Towards Cooperation by Carrier Aggregation in Heterogeneous Networks: A Hierarchical Game Approach

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***Abstract*—This paper studies the resource allocation problem for a heterogeneous network (HetNet) in which the spectrum owned by a macro-cell operator (MCO) can be shared by both unlicensed users (UUs) and licensed users (LUs). We formulate a novel hierarchical game theoretic framework to jointly optimize the transmit powers and sub-band allocations of the UUs as well as the pricing strategies of the MCO. In our framework, an overlapping coalition formation (OCF) game has been introduced to model the cooperative behaviors of the UUs. We then integrate this OCF game into a Stackelberg game-based hierarchical framework. We prove that the core of our proposed OCF game is non-empty and introduce an optimal sub-band allocation scheme for UUs. A simple distributed algorithm is proposed for UUs to autonomously form an optimal coalition formation structure. The Stackelberg Equilibrium (SE) of the proposed hierarchical game is derived and its uniqueness and optimality are proved. A distributed joint optimization algorithm is also proposed to approach the SE of the game with limited information exchanges between the MCO and the UU.**

I. Introduction

A HetNet is a multiple tier network consisting of co-located macro-cells, micro-cells and femto-cells. It has been included in the Long Term Evolution Advanced (Linto9031.00h0h.b90.91((L)9Tluded

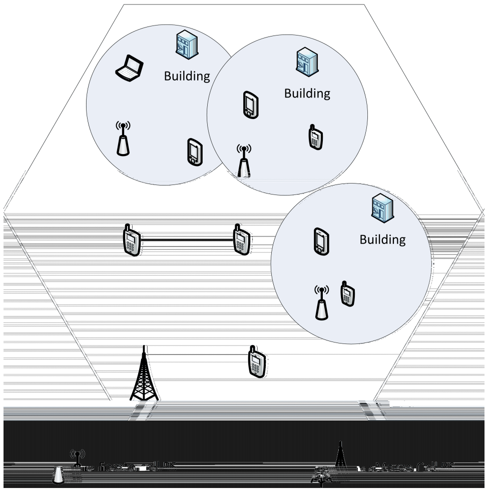

Fig. 1. A spectrum sharing multi-tier HetNet in which the spectrum is owned by the macro-cell and shared with other tiers.

algorithm that can approach the structure that is in the core of our proposed game. The main contributions of this paper are summarized as follows:

1) A spectrum-sharing based HetNet is considered in which

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the game. In [24], the authors investigated the cooperative behaviors of secondary users in a two-tier spectrum sharing cognitive network where both the Stackelberg game and non-overlapping coalition formation game were combined to build a hierarchical game framework. A joint solution was given to the sub-band allocation and interference control problem. Although the coaltional game has been widely used to study the problems in wireless communications, most of the existing works only allow users to form disjoint coalitions. In practical communication systems, allowing overlapping of coalitions can further improve the performance [25]. For example, one mobile subscriber may cooperatively transmit in two different sub-bands with two different subscribers. However, so far only limited works have been reported to apply the overlapping coalitional game to analyze cellular networking systems. In [31], investigation was made on how small cell BSs coordinate with each other to achieve efficient transmission. By allowing the femto-cells to form overlapping coalitions to jointly schedule the transmission of their subscribers, it was found that the performance of mobile nodes located at the edge of the coverage areas can be further improved. One of the key differences between the proposed work and the previously reported results is that we adopt a new OCF-game model which enables each player to join multiple coalitions.

III. System Setup

Consider an orthogonal frequency-division multiple access (OFDMA) based two-tier network where the spectrum owned by an MCO is divided into M sub-bands. Each of the sub-bands can be accessed by multiple UUs controlled by the femto-cell BSs as illustrated in Fig. 1. We denote the set of sub-bands as $\mathcal{B}$ and the set of femto-cell BSs as $\mathcal{K}$. Here the concept of *underlay* is borrowed from the cognitive radio which means that each secondary user (i.e., UU) is allowed to access the spectrum of primary users (i.e., LUs) which can tolerate limited interference from the UUs [33]. In this paper, we consider frequency selective fading, i.e., channel fading in different sub-bands is independent. We assume the channel state is time-invariant and can be regarded as a constant within each time slot. It is also assumed that the mobile devices are equipped with multiple antennas and hence can transmit over multiple sub-bands at the same time. Furthermore, multiple UUs are allowed to share the same sub-band with UUs. The system analysis is performed by using numerical calculations and the simulation on Matlab platform.

Each femto-cell BS can apply multiple sub-bands to support services for UUs, i.e., each sub-band can be accessed by the UUs from more than one femto-cell BS. We assume that in each time slot there is only one active UU S_k connected with femto-cell BS k. Let h_k^m be the channel gain between S_k and the macro-cell BS receiver in sub-band m, and g_{kj}^m be the channel gain between S_k and jth femto-cell BS. Let $\boldsymbol{p}_{S_k} = [p_{S_k}^1, ..., p_{S_k}^M]$ be the power allocation vector of UUs, where $p_{S_k}^m = 0$ implies that sub-band m is not used by S_k. Table I lists the notations and symbols used in this paper. Multiple femto-cell BSs can apply for the same sub-band at the same time. We denote the set of all UUs as $\mathcal{S}$ and the set of UUs utilizing the same sub-band m as $\mathcal{L}_m$, i.e., $\mathcal{L}_m = \{S_k : p_{S_k}^m > 0, \forall S_k \in \mathcal{S}\}$. $\mathcal{L}_m = \emptyset$ meameans 330.00109(no1.49Td[6.8820Td()T455(sub

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generally different in different sub-bands. Hence the UUs are preferred to transmit in those frequency bands with weak channel gains between the UUs and the macro-cell BS.

An important problem is how UUs can distributively form different coalitions to improve their pay-offs. We formulate an overlapping coalition formation game to study this problem. In this game, UUs can behave cooperatively to coordinate their actions. Hence the coalition formation game focuses on solving the following two questions: a) how the coalition members coordinate with each other, and b) how a coalition formation structure can be established among UUs.

To answer the first question, the virtual MIMO technique is used as the cooperation scheme among the UUs in the same coalition because it is shown to achieve the upper-bound of the rate for a multiple access channel [21], and to satisfy the proportional fairness [24]. More specifically, the UUs in the same sub-band m form a coalition and cooperate with each other to transmit and receive signal. Using the virtual MIMO technique, we can convert the communication within one coalition into a virtual $\mathcal{L}_m$-input $\mathcal{L}_m$-output channel, which follows the same line as [24] and [21]. Therefore the capacity sum of all UUs in the mth virtual MIMO channel is obtained as,

$$\sum_{S_k\in\mathcal{L}_m} r_{S_k} = \sum_{S_k\in\mathcal{L}_m} \log\left(1+\lambda^m_{S_k} p^m_{S_k}\right), \tag{3}$$

where $\lambda^m_{S_k}$ is the kth non-zero eigenvalue of matrix $\boldsymbol{G}^T_{\{S_k\in\mathcal{L}_m\}}\boldsymbol{G}_{\{S_k\in\mathcal{L}_m\}}$ where $\boldsymbol{G}_{\{S_k\in\mathcal{L}_m\}}$ is the channel gain matrix of UUs in the same sub-band. For example, if $\{S_1,...,S_n\}$ are in the same sub-band m, then the matrix is given by

$$\boldsymbol{G}_{\{S_k\in\mathcal{L}_m\}} = \begin{bmatrix} g^m_{11} & g^m_{12} & \cdots & g^m_{1n} \\ g^m_{21} & g^m_{22} & \cdots & g^m_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ g^m_{n1} & g^m_{n2} & \cdots & g^m_{nn} \end{bmatrix}. \tag{4}$$

In the above matrix, $g^m_{jk} = \frac{g^{m\prime}_{jk}}{\sigma^m_k}$, where $g^{m\prime}_{jk}$ is the channel gain between UU S_j and femto-cell BS k, and σ^m_k is the received interference power at BS k in sub-band m. Note that as σ^m_k changes, the action of the UU adjusts adaptively, hence the negative externality brought by the inter-cell interference is compensated. We will give detailed analysis and propose a distributive algorithm to answer the second question in Section V.

To simplify the analysis, let us consider the uplink transmission. In the uplink, the receiver of macro-cell BS is interfered by the transmit signals of UUs. Therefore there is only one leader when it applies price-based interference control. However, our model can be directly extended to the downlink scenario. In the downlink case, multiple LUs act as a group of leaders which can cooperatively decide the interference price in each sub-band. The main objectives of this paper are to solve the following problems:

1) *Power control problem*: investigating how the MCO controls the interference power to protect the LUs by dynamically adjusting the interference price.
2) *Sub-band allocation problem*: investigating how the UUs choose the sub-bands to access based on the channel information, the interference price and the action of other UUs.
3) *Overlapping Coalition formation problem*: investigating how the UUs form overlapping coalitions to improve their data rate.

A hierarchical game framework is formulated to jointly optimize the solutions to above three problems.

IV. The Hierarchical Game Formulation

The interaction between the macro-cell BS and femto-cell BS can be modeled as a Stackelberg game. Furthermore, we also formulate an OCF-game to investigate the cooperation among the femto-cell BSs, where their UUs can form coalitions to improve the performance. It is assumed that the transmission of femto-cell and macro-cell is synchronized.

Let us jointly solve the power control problem of the LUs and resource allocation problem of the UUs. Firstly, there is a trade-off between the capacity sum of the femto-cell network and QoS of the macro-cell. If the UUs transmit with high power, they will get high data rate but generate more interference to the macro-cell BS. Since sufficient protection to the LUs should be guaranteed in the first place, the MCO should regulate the behavior of the UUs, which can be modeled as a power control problem for UUs. Secondly, given the limited spectrum and power resources, we should consider how the UUs can cooperate with each other to allocate the sub-band and optimize power consumption.

Let us consider a hierarchical game consisting of the two sub games. In the proposed game model, the MCO and femto-cell BSs are the *players*. The way the players play the game is defined as *actions*. The action of the MCO is to decide the interference prices, and the actions of the UUs are to decide which sub-bands to access and how much power should be allocated to each of these sub-bands.

The Stackelberg game is used to model the interaction between the MCO and the femto-cell BSs. In the proposed Stackelberg game, the *leader* is the MCO and the corresponding LUs and the *followers* are the femto-cell BSs who control the UUs. Let us follow a commonly adopted game theoretic setup [10] [23] [20] to define the pay-off of S_k as,

$$\pi_{S_k}(\boldsymbol{p}_{S_k},\boldsymbol{\mu}) = r_{S_k}(\boldsymbol{p}_{S_k}) - c_{S_k}(\boldsymbol{p}_{S_k},\boldsymbol{\mu}), \tag{5}$$

where $c_{S_k}(p_{S_k,\mu}) = \sum_{m=1}^{M}\mu^m h^m_{S_k} p^m_{S_k}$ is the cost function. Furthermore, since S_k can simultaneously access multiple sub-bands, it aims to maximize the sum of the pay-offs obtained from all the active sub-bands under the constraints given in (1) and (2).

The MCO collects the payment from all the UUs occupying the sub-bands and the pay-off functions of the MCO are defined as,

$$\pi_{MCO}(\boldsymbol{p}_{S_k},\boldsymbol{\mu}) = \sum_{k=1}^{S_k} c_{S_k}(\boldsymbol{p}_{S_k},\boldsymbol{\mu}). \tag{6}$$

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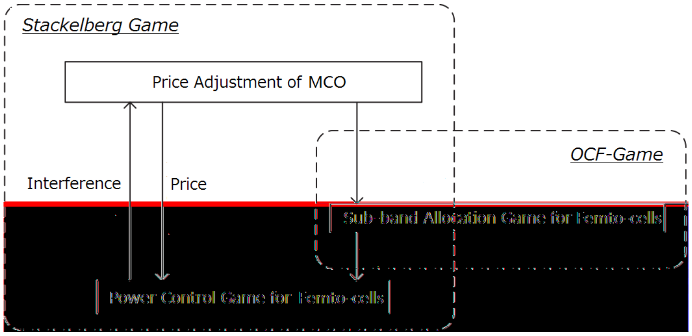

Fig. 2. The hierarchical game structure.

The main solution to our proposed hierarchical game is the Stackelberg equilibrium (SE) which is formally defined as follows [3]:

Definition 1. *For a fixed sub-band allocation, the pricing vector* $\boldsymbol{\mu}^* = [\mu_1^*, ..., \mu_M^*]$ *and the transmit power* $\boldsymbol{p}_{S_k}^* = [p_{S_k}^{1*}, ..., p_{S_k}^{M*}], k = 1, ..., K$*, form a SE if the interference power constraint in (1) is satisfied, and for any* $m \in \{1, ..., M\}$ *and* $k \in \{1, ..., K\}$*, we have*

$$\mu_m^* = \arg\max_{\mu_m \geq 0} \pi_{MCO}(\boldsymbol{p}^*, \mu_m, \mu_{-m}^*) \tag{7}$$

where μ_{-m} *means all the MCO except for* m*. For any given price* $\boldsymbol{\mu}$*,* $\boldsymbol{p}^*$ *is given by*

$$\boldsymbol{p}_{S_k}^* = \arg\max_{p_{S_k} \geq 0} \pi_{S_k}(p_{S_k}, \boldsymbol{p}_{-S_k}^*). \tag{8}$$

The structure of the hierarchical game is illustrated in Fig. 2. The MCOs can adjust their prices to maximize the pay-off defined in (6). We will show that the optimal price is specified by the dynamics of the interference from the CA in each sub-band. The femto-cells BSs can cooperate and self-organize into coalitions, each of which consists of member UUs to coordinate the transmission to improve the sum of the pay-offs. On the femto-cells BS side, they cooperate and self-organize into coalitions, in which their member UUs can coordinate their transmission to improve the sum of pay-off.

A. The pay-off of UU

Suppose that the overlapping coalition formation structure is fixed and each S_k has already obtained a fixed λ_{S_k}. We can write the payoff of each UU S_k as

$$\pi_{S_k}^m(p_{S_k}^m, \mu^m, \lambda_{S_k}^m) = \log\left(1 + \lambda_{S_k}^m p_{S_k}\right) - \mu^m h_{S_k}^m p_{S_k}^m. \tag{9}$$

The optimal power allocation of S_k is obtained by solving the following optimization problem,

Problem 1.

$$\max_{\boldsymbol{p}_{S_k}} \pi_{S_k}(\boldsymbol{p}_{S_k}, \boldsymbol{p}_{-S_k}, \boldsymbol{\mu}, \boldsymbol{\lambda}_{S_k})$$
$$\text{S.t.} \quad \sum_{m=1}^{M} p_{S_k}^m \leq \overline{p}.$$

In the proposed Stackelberg game framework, the maximum tolerable interference in (1) is omitted in Problem 1 because it is included in the interference μ^m and thus is always satisfied. Hence we only need to consider the constraint in (2).

Problem 1 can be directly solved by using the standard convex optimization approaches and the resulting optimal transmit power for UU S_k in sub-band m is given by,

$$p_{S_k}^{m\dagger} = \arg\max_{p_{S_k}^m \geq 0} \pi_{S_k}^m(p_{S_k}^m, \mu^m, \lambda_{S_k}^m) \tag{10}$$
$$= \left(\frac{1}{\mu^m h_{S_k}^m} - \frac{1}{\lambda_{S_k}^m}\right)^+. \tag{11}$$

Let $\boldsymbol{p}_{S_k}^\dagger = [p_{S_k}^{1\dagger}, p_{S_k}^{2\dagger}, ..., p_{S_k}^{m\dagger}]$. Due to the power cap constraint in (2), the final power allocation will fall into the following two cases:

Case 1. *If* $\sum_{m=1}^{M} p_{S_k}^{m\dagger} \leq \overline{p}$*: In this case,* S_k *can access all sub-bands under the constraint defined in (2). The power allocation of* S_k *is decided by constraint in (1). Hence we can remove (2) and the power allocation of the UU solely depends on the sub-band prices. Each of the UUs tries to solve (9) for the optimal power allocation and obtain* $p_{S_k}^{m\dagger}$ *to maximize the pay-off.*

Case 2. *If* $\sum_{m=1}^{M} p_{S_k}^{m\dagger} > \overline{p}$*: In this case, only selected sub-bands can be accessed by the UU* S_k*. More specifically, the solution is achieved by searching a sub-set* $\mathcal{N}_i \subset \mathcal{M}$ *such that the following condition is satisfied:*

$$\sum_{m \in \mathcal{N}_i} \pi(p_{S_k}^{m\dagger}) \geq \sum_{n \in \mathcal{N}_j, j \neq i} \pi(p_{S_k}^{n\dagger}), \tag{12}$$

where $\{\mathcal{N}_j\}$ *denotes the set of all possible sub-sets of* $\mathcal{M}$ *except* $\mathcal{N}_i$*. This case implies that once the price is fixed, the number of sub-bands accessed by one UU is bounded by the power cap constraint, and obviously we have* $\sum\limits_{m \in \mathcal{N}_i} p_{S_k}^{m\dagger} \leq \overline{p}$*.*

In either case, we can obtain the optimal power allocation of S_k:

$$\boldsymbol{p}_{S_k}^* = \{p_{S_k}^{m*}, m = 1, 2, ..., M.\},$$
$$p_{S_k}^{m*} = \begin{cases} p^{m\dagger}, \text{if } m \in \mathcal{N}_i \\ 0, \text{otherwise.} \end{cases} \tag{13}$$

The corresponding sub-band allocation indicator is

$$\boldsymbol{l}_{S_k}^* = \{l_{S_k}^{m*}, m = 1, 2, ..., M.\},$$
$$l_{S_k}^{m*} = \begin{cases} 1, \text{if } \boldsymbol{l}_{S_k}^* > 0, \\ 0, \text{otherwise.} \end{cases} \tag{14}$$

From the results above, it can be observed that the optimal solution to the transmit power only depends on the values of μ^m and $\lambda_{S_k}^m$. The prices are decided by the MCO through its interaction with UUs, and $\lambda_{S_k}^m$ is obtained from coalition formation structures of UUs. In the rest of this section, we discuss how to obtain optimal μ^m and $\lambda_{S_k}^m$.

B. The pay-off of the MCO

The MCO can use the prices $\boldsymbol{\mu}$ charged to the UUs to control the interference in each sub-band. We will show that the MCO can maximize its pay-off by adjusting the prices based on the dynamic of the aggregated interference at the macro-cell BS receiver. Hence the proposed algorithm greatly reduces the communication overhead and makes the distributed power allocation approach possible.

The revenue gained by the MCO by sharing sub-band m is given by:

$$\pi_{MCO}(\boldsymbol{p}^m,\mu^m)=\mu^m\sum_{k=1}^{K}h^m_{S_k}p^m_{S_k}. \tag{15}$$

Hence the MCO tries to find the optimal sub-band price to maximize its revenue in each sub-band under the maximum tolerable interference constraint.

Problem 2.

$$\max_{\mu^m}\ \pi_{MCO}(\boldsymbol{p}^m,\mu^m) \tag{16}$$

$$\text{s.t.}\ \sum_{k=1}^{K}p^m_{S_k}h^m_{S_k}\leq\overline{Q}. \tag{17}$$

$$p^m_{S_k}\geq 0. \tag{18}$$

Substitute (11) into Problem 2, we obtain

Problem 3.

$$\max_{\boldsymbol{\mu}}\ \sum_{k=1}^{K}\left(\frac{1}{h^m_{S_k}}-\frac{\mu^m}{\lambda^m_{S_k}}\right)^+ h^m_{S_k} \tag{19}$$

$$\text{s.t.}\ \sum_{k=1}^{K}\left(\frac{1}{\mu^m h^m_{S_k}}-\frac{1}{\lambda^m_{S_k}}\right)^+ h^m_{S_k}\leq\overline{Q}. \tag{20}$$

Using standard convex optimization approach to find the optimal μ^m in above problem requires the MCO to obtain global information of the UUs. Fortunately, Problem 3 has a nice property that the objective and constraint functions both monotonically decrease with μ^m. Hence if we assume the power cap constraint is satisfied, then the objective function will be maximized when the constraint in (20) takes equality. Note that the left side of (20) is the aggregated interference received by macro-cell BS in sub-band m. Therefore the MCO can optimize price μ^m and affect the aggregated interferences to the upper bound.

V. Coalition Formation Game Analysis

In this section, we first define the coalitional game and imputation, and then analyze the game properties to prove the existence of the core.

Definition 2 ([18], Chapter 9). *A coalition $\mathcal{C}$ is a non-empty sub-set of the set of all players $\mathcal{K}$, i.e., $\mathcal{C}\subseteq\mathcal{K}$. A coalition of all players is referred as the grand coalition $\mathcal{K}$. A coalitional game is defined as $(\mathcal{C},v)$ where v is the value function mapping a coalition structure $\mathcal{C}$ to a real value $v(\mathcal{C})$. A coalitional game is said to be super-additive if for any two disjoint coalitions $\mathcal{C}_1$ and $\mathcal{C}_2$, $\mathcal{C}_1\cap\mathcal{C}_2=\emptyset$ and $\mathcal{C}_1,\mathcal{C}_2\subset\mathcal{K}$, we have,*

$$v(\mathcal{C}_1\cup\mathcal{C}_2)\geq v(\mathcal{C}_1)+v(\mathcal{C}_2). \tag{21}$$

Given two coalitions $\mathcal{C}_1$ and $\mathcal{C}_2$, we say $\mathcal{C}_1$ and $\mathcal{C}_2$ overlap if $\mathcal{C}_1\cap\mathcal{C}_2\neq\emptyset$.

Definition 3. *A pay-off vector $\boldsymbol{\pi}$ is a division of the value $v(\mathcal{C})$ to all the coalition members, i.e., $\boldsymbol{\pi}=[\pi_{S_1},\cdots,\pi_{S_K}]$. We say $\boldsymbol{\pi}$ is group rational if $\sum_{k=1}^{K}\pi_{S_k}=v(\mathcal{C})$ and individual rational if $\pi_{S_k}\geq v(\{S_k\}),\forall S_k\in\mathcal{C}$. We define an imputation as a pay-off vector satisfying both group and individual rationalities.*

If a coalitional game satisfies the supper-additive condition, all the players will have the incentive to form a grand coalition. However if the supper-additive condition does not hold, then the grand coalition will not be the optimal solution for all players. In this case, the players will try to form a stable coalition formation structure in which no player can profitably deviate from it. In the proposed OCF-game, for each possible prices of the MCO, we focus on finding optimal coalition formation structure for UUs to share the spectrum of the MCO.

When overlapping is enabled among coalitions, the coalitions are no longer disjoint sub sets of the player set as defined in the non-overlapping coalitional game. In the OCF-game, the concept *partial coalition* is utilized.

Definition 4. *The partial coalition is defined as a vector $\boldsymbol{p}^m=(p^m_{S_1},p^m_{S_2},...,p^m_{S_K})$, where $p^m_{S_k}$ is the fractional resource of S_k dedicated to coalition m. Note that $p^m_{S_k}=0$ means S_k is not a member of the mth coalition. A coalition structure is a collection $\boldsymbol{P}=(\boldsymbol{p}^1,...,\boldsymbol{p}^M)$ of partial coalitions.*

Remark 1. *In a non-overlapping coalition formation game, a coalition is just a subset of the player set. For a player set of size N, the number of coalition formation structures is given by the Bell number B_N, where $B_N=\sum_{k=0}^{N-1}\binom{N-1}{k}B_k$ is the number of possible coalition structures and B_k is the number of ways to partition the set into k items.*

For example, for a game with two players S_1 and S_2, the possible partitions can be written as $\{S_1,S_2\}$ or $\{\{S_1\},\{S_2\}\}$. However, in OCF-game the concept of partial coalition not only specifies who joins each coalition, but also indicates how much resource each player will allocate to each coalition. If the resource is continuous, there are generally an infinite number of partial coalitions. It means that the concept of coalition can be regarded as a special case of the partial coalition, where each player joins only one coalition with all its resource.

Definition 5. *An OCF-game is denoted by $G=(\mathcal{K},\mathcal{M},\boldsymbol{P},\boldsymbol{v})$, where*

- *$\mathcal{K}=\{1,2,...,K\}$ is the set of players which are the femto-cell BSs.*
- *$\mathcal{M}=\{1,2,...,M\}$ is the set of sub-bands.*
- *$\boldsymbol{P}$ is the power allocation matrix, where the row $\mathbf{p}_{S_k}=(p^1_{S_k},p^2_{S_k},...,p^M_{S_k})$ represents how player S_k assigns its power on different sub-bands, and the column $\boldsymbol{p}^m=(p^m_{S_1},p^m_{S_2},...,p^m_{S_K})$ represents the power each player consumes for sub-band m. $\boldsymbol{p}^m=(p^m_{S_1},p^m_{S_2},...,p^m_{S_K})$ also corresponds to a partial coalition.*
- *$\boldsymbol{v}(\boldsymbol{C}^m):\mathbb{R}^n\longrightarrow\mathbb{R}^+$ is the value function, which represents the total pay-off of a partial coalition $\boldsymbol{C}^m$.*

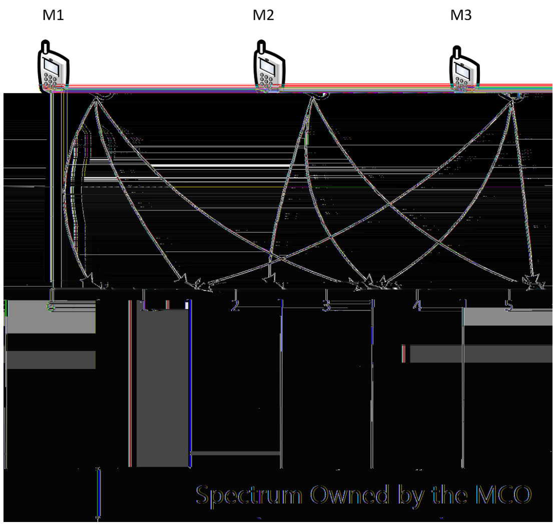

Fig. 3. The illustration of the overlapping coalitions in our proposed game.

Definition 6. *We define a game to be U-finite if for any coalition structure that arises in this game, the number of all possible partial coalitions is bounded by U.*

Fig. 3 illustrates an example of the overlapping coalition formation of our model. Suppose the spectrum of the MCO is consists of six sub-bands $\{1,2,3,4,5,6\}$ which can be allocated to three mobile devices $\{M1,M2,M3\}$. A coalition is formed by two or more mobile devices accessing the same sub-band. Each mobile device may belong to multiple coalitions, since it may access multiple sub-bands at the same time. The coalitions containing a common member player overlap. In Fig. 3, for example, we denote the coalition formed by the devices accessing sub-band k as $\mathcal{C}_k$. We have $\mathcal{C}_1 = \{M1\}$, $\mathcal{C}_2 = \{M1,M3\}$, $\mathcal{C}_3 = \{M3\}$, $\mathcal{C}_4 = \{M1,M2,M3\}$, $\mathcal{C}_6 = \{M2,M3\}$, $\mathcal{C}_5 = \emptyset$. Hence, $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_4$ overlap with each other since $\mathcal{C}_1 \cap \mathcal{C}_2 \cap \mathcal{C}_4 = \{M1\}$. Similarly, $\mathcal{C}_3 \cap \mathcal{C}_4 \cap \mathcal{C}_6 = \{M2\}$ and $\mathcal{C}_2 \cap \mathcal{C}_4 \cap \mathcal{C}_6 = \{M3\}$.

The sum rate achieved by forming coalition is given by (3), and the pay-off sum of UUs equals to the sum rate minus the payment to the MCO. Hence the value function of the partial coalition $\boldsymbol{p}^m$ is defined as the pay-off sum on sub-band m. Given the fixed price vector $\boldsymbol{\mu}$, the value function of the partial coalition $\boldsymbol{p}^m$ is given by,

$$\boldsymbol{v}(\boldsymbol{p}^m, \boldsymbol{\lambda}^m) = \sum_{S_k \in \mathcal{L}_m} r_{S_k} - \sum_{S_k \in \mathcal{L}_m} \mu^m h^m_{S_k} p^m_{S_k}. \tag{22}$$

It is proved in [24] that the pay-off division among coalition members satisfies the proportional fairness [12] and if the benefit allocated to each member equals to its contribution to the overall rate in sub-band m, i.e.,

$$r^m_{S_k} = \log\left(1 + \lambda^m_{S_k} p^m_{S_k}\right). \tag{23}$$

The solution of the optimal power vector $p^m_{S_k}$ of S_k is given by (13), which is a function of $\lambda^m_{S_k}$ and μ^m. Since $\boldsymbol{\mu}$ is imposed by the MCO, the UUs can optimize their pay-off sum by choosing proper sub-bands to access. Furthermore, since $\lambda^m_{S_k}$ is decided by the coalition structure, finding sub-band allocation will directly affect the payoff of each UU.

There are two types of actions in an OCF-game, which are the coalitional action and the overlapping action. The former defines how the resource being allocated among the member players in one coalition, and the latter defines how resources being allocated between players in the overlapping parts of multiple coalitions. These are the key features to differentiate the OCF-game from the non-overlapping coalition formation game.

In the proposed system setup, the femto-cell BSs whose UUs are accessing the same sub-band form a coalition. The cooperation among the member players is achieved by forming a virtual MIMO channel. The pay-off division relies on assigning $\boldsymbol{\lambda}$ to the players, which can be considered as the contribution of each coalition member to the sum rate. Since the UUs can join multiple coalitions, the proposed game becomes an OCF-game. The resource of a UU includes its total transmit power. The UUs need to allocate its transmit power in each sub-band properly for maximizing the pay-off. For the proposed OCF-game, we have the following definition.

Definition 7. *For a set of UUs $\mathcal{S}$, a coalition structure on $\mathcal{S}$ is a finite list of vectors (partial coalitions) $\boldsymbol{P} = (\boldsymbol{p}^1, ..., \boldsymbol{p}^M)$ that satisfies (i) $\sum_{k=1}^{K} h^m_{S_k} p^m_{S_k} \leq \overline{Q}$, (ii) $\sup \boldsymbol{p}^m \subseteq \mathcal{S}$ for all $m = 1, ..., M$, and (iii) $\sum_{m=1}^{M} p^m_{S_k} \leq \overline{p}$ for all $j \in \mathcal{S}$.*

The power allocation matrix also indicates the utilization status of sub-bands. The constraint (i) states that the transmit power of UU in each sub-band is bounded, (ii) states that the

Since enabling overlapping in the coalition formation game will significantly increase the complexity of the game, the overlapping coalition structure is sometimes unstable as there may exist cycles in the game play. For example, let us consider a network system with three UUs S_1, S_2 and S_3, and two sub-bands l_1 and l_2. We denote $\pi_{S_j}[m|S_i]$ as the pay-off obtained by S_j when it forms coalition with S_i on sub-band m, and $\pi_{S_j}[m|\emptyset]$ is the pay-off obtained by S_j when it exclusively occupies m. Initially, since $\pi_{S_1}[l_1|\emptyset] > \pi_{S_1}[l_2|\emptyset]$, $\pi_{S_2}[l_2|\emptyset] > \pi_{S_2}[l_1|\emptyset]$ and $\pi_{S_3}[l_2|\emptyset] > \pi_{S_3}[l_1|\emptyset]$, S_1 joins l_1, S_2 and S_3 join l_2 . 3

bound of v_{S_k} and $\underline{h}$ be the lower bound of $|h_{jk}|^2$, then we have $\overline{\mu} = \frac{\overline{v}}{\underline{h}}$.

Algorithm 1 OCF Algorithm for Sub-band Allocation

Step - 1) *Sensing:*

a) The UUs, after receiving the prices of available sub-bands from the MCO, sequentially send a short training message to estimate their pay-off in all the sub-bands when the sub-bands are exclusively used by S_k.

b) Each S_k broadcasts the sub-band combination $\boldsymbol{l}^*_{S_k}$ that maximizes its pay-off sum,

$$\boldsymbol{l}^*_{S_k} = [l^{(1)}_{S_k}, l^{(2)}_{S_k}, ..., l^{(n)}_{S_k}]. \tag{25}$$

Let $\mathcal{R}^* = \{\boldsymbol{l}^*_{S_k} : S_k \in \{1, ..., K\}\}$.

Step - 2) *Negotiation:*

a) All the active UUs need to negotiate with each other on each of the sub-bands in $\mathcal{R}^*$ to obtain the possible pay-off division factor $\lambda^m_{S_k}$.

b) After the negotiation process, S_k solves problem (1) based on the new set of $\lambda^m_{S_k}$, and obtains a new sub-band allocation to maximize its pay-off. Then S_k updates and broadcasts its optimal sub-bands allocation. Step 2) is repeated until no UU wants to change its occupied sub-bands.

Algorithm 2 Distributed Interference Control Algorithm

Definitions: At iteration t, let

- $\mu^m(t)$ be the pricing coefficient of sub-band m,

Step - 1) *Initialization:*

- Set $\mu^m \geq \overline{\mu}, \forall m \in \{1, 2, ..., M\}$.
- Set $\epsilon > 0$ to be a small positive constant.

Step - 2) *Price Adjustment:*

a) At iteration t, MCO updates and broadcasts $\boldsymbol{\mu}(t) = (1 - \epsilon)\boldsymbol{\mu}(t-1)$.

b) Each S_k senses the sub-bands and negotiates with other active UUs in the same sub-bands to determine the sub-band allocation $\boldsymbol{l}^{m*}(t)$ and power allocation $\boldsymbol{p}^{m*}(t)$.

c) All active UUs repeat Step 2-b) to update their optimal sub-bands, and the outcome is a coalition structure $\boldsymbol{P}^m(t)$.

d) The MCO monitors the aggregated interference in each sub-band. If $N_j > \overline{Q}$, the price adjustment in sub-band j stops. If $N_j \leq \overline{Q}$, go to Step 2a).

Step - 3) *Termination:*

The algorithm ends with solution $\boldsymbol{\mu}^* = \boldsymbol{\mu}(t-1), \boldsymbol{P}^* = \boldsymbol{P}(t-1)$ in which the element $p^{m*}_{S_k}(\mu^{m*})$ is given by (13).

Algorithms I and II are proposed to find the SE of the hierarchical game. For any given $\overline{Q}, \overline{p}$ pair and the channel gains, the algorithms achieve the SE which contains a stable overlapping coalition structure and an optimized power allocation for each UU. We have the following proposition about the SE of the game.

Proposition 3. *The price μ^m always converges to a non-negative value if a non-negative power allocation for a given $\overline{p}$ and $\overline{Q}$ pair exists.*

Proof 3. *: See Appendix C.* □

From propositions 2 and 3, we conclude that, for any given $\overline{p}$ and $\overline{Q}$, the proposed algorithms will converge to the SE of the hierarchical game. The simulation results provided in Section VII support this claim.

Remark 2. *The hierarchical game works as follows. At the beginning of iteration, the MCO broadcasts the price $\boldsymbol{\mu}$ to all UUs in its coverage area. Each UU decides its optimal transmit power and sub-band based on the received pricing information sent by MCO. Once all UUs have made the decisions, MCO will adjust the price based on the interference before going to the next iteration.*

The proposed algorithms can be implemented in a distributed manner. On the MCO side, it does not need any information from the UUs, e.g., the interfering link gain $h^m_{S_k}$ or corresponding transmit power $p^m_{S_k}$. It simply measures the aggregated interference at its receiver in each channel, and adjusts the price accordingly. On the UUs side, with the channel price and the link gain information measured within a coalition, they can easily derive the potential pay-off gained by joining different coalitions. Therefore each of them can choose the coalition that maximizes its payoff to join.

Considering the time overhead for information exchange between the MCO and the UUs, there is a need for only one dedicated channel for the MCO to broadcast the interference prices. The implementation is illustrated in Fig. 4. A time frame for data transmission can be divided into two phases: the power control phase and the data transmission phase. The power control phase is divided into several time slots, which corresponds to an iteration in the proposed interference control algorithm. In each time slot, the MCO first measures the interference it is suffering, then adjusts the interference price in each sub-band. Upon receiving the interference prices, the UUs re-allocate their power in each sub-band based on the prices and the measured mutual interference. After several iterations when the prices and power allocation are stable, each of the UUs uses its power allocation in the last time slot to perform data transmission. Supposing the price and power allocation converge after L time slots, each time slot duration is τ, and the data transmission time is t, the time overhead of the proposed algorithm is given by $\frac{t}{K\tau + t}$.

VII. Numerical Results

In this section we investigate the performance of the proposed hierarchical game framework in the spectrum-sharing based femto-cell network. To better illustrate how to apply the proposed algorithm to various network environments, we consider the network system under different sets of interference and power constraints, as well as different numbers of UUs K and available sub-bands M combinations.

Fig. 5 illustrates the convergence of interference in a network with 8 sub-bands, $\overline{p} = 50$ and $\overline{Q} = 2$. It is noted that the prices in each sub-band converge at the similar speeds.

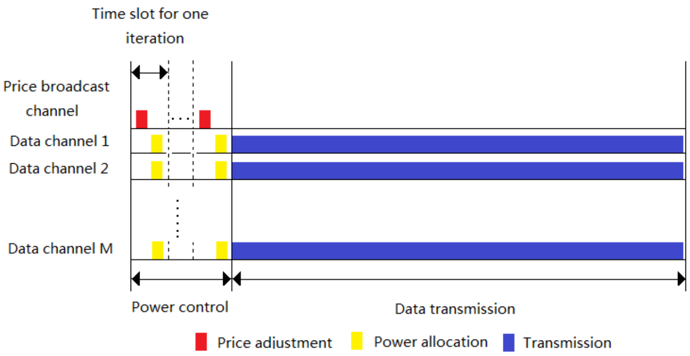

Fig. 4. A time frame of proposed algorithm.

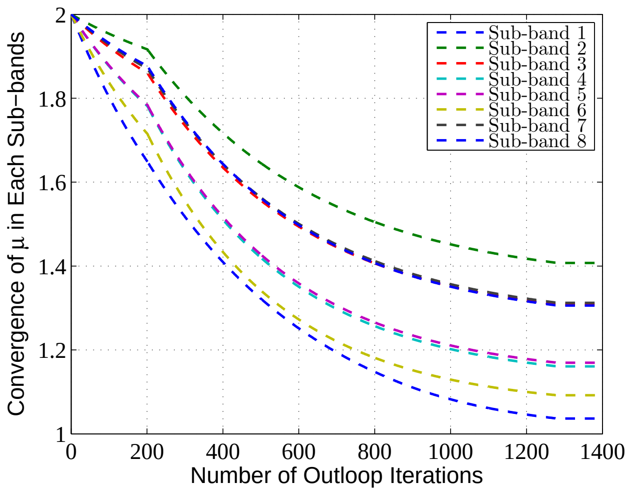

Fig. 5. Convergence performance. $\overline{p} = 50$, $\overline{Q} = 2$. The curves illustrate the convergence of the interference prices in an 8 sub-band network.

This is because the prices of MCO directly control sub-band allocation and the power allocation of UUs. Finally, the prices charged to different sub-bands are independent with each other, which coincides with the definition in (18).

In Fig. 6, the convergence rate of average prices for different $\overline{Q}$ values is provided. An interesting observation is that, under the same power cap constraint, the convergence speed in the case of large $\overline{Q}$ is generally much faster than that in the case of small $\overline{Q}$. This phenomenon can be explained as follows. With the increase of $\overline{Q}$, each UU will allocate more power in each sub-band. Hence under a fixed power cap constraint, each UU can access fewer sub-bands or, equivalently, join few coalitions. Hence, a large $\overline{Q}$ reduces the chance for UUs to join many coalitions, which results in a reduced complexity for coalition formation. Thus the time cost on forming a stable coalition structure can be significantly reduced.

Figs. 7 to 8 show the convergence rate of the sub-band prices as well as the pay-offs of the MCO and UUs network. The tested network contains 64 UUs and 128 sub-bands, with $\overline{p} = 100$. Fig.7 compares the pay-offs of the MCO versus the interference and power constraints. Assuming the channel coefficients are fixed, we increase one constraint while fixing the others. It is observed that at the beginning of each time slot, the pay-offs increase with the constraint before

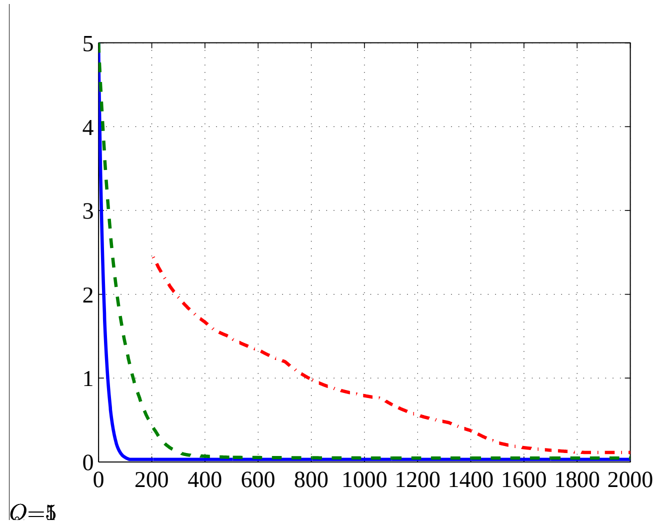

Fig. 6. Convergence performance of the price under different $\overline{Q}$, $\overline{p} = 100$. The curve shows the impact of $\overline{Q}$ on the convergence speed.

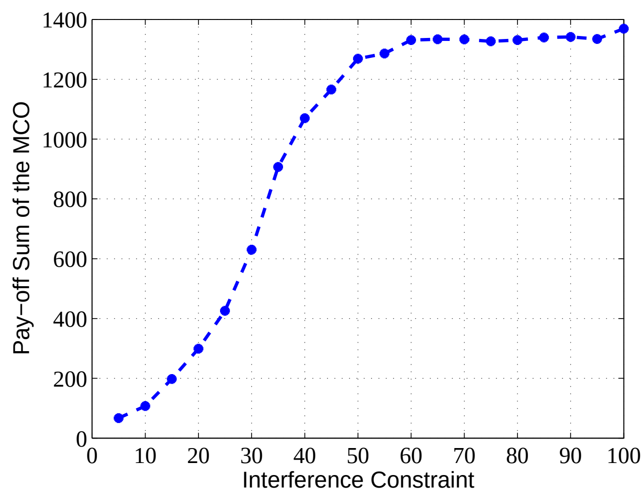

Fig. 7. The impacts of varying the interference constraint: the pay-off sum increases with $\overline{Q}$.

they become steady. The reason for this is that initially the interference constraint is much tighter, which becomes the main limitation of the transmit power. However, when the interference constraint becomes larger, the transmit power is then jointly limited by both interference and power cap constraints. Finally when the interference constraint becomes very loose, the transmit power is limited by the power cap constraint so the system performance becomes stable.

Fig. 8 illustrates the choice of interference limit $\overline{Q}$ against the average price $\overline{\mu}$ over all sub-bands. The average price $\overline{\mu}$ generally reflects how much interference LUs can tolerate. It is observed that the price at $\overline{Q} = 10$ is higher than that at $\overline{Q} = 50$. This shows that the price decreases with the value of $\overline{Q}$. Because the smaller the value of $\overline{Q}$ means the rarer of the resource, the price is accordingly increased. More specifically, it is obvious that the larger the $\overline{Q}$, the larger the possible transmit power of UU. Considering the optimal power

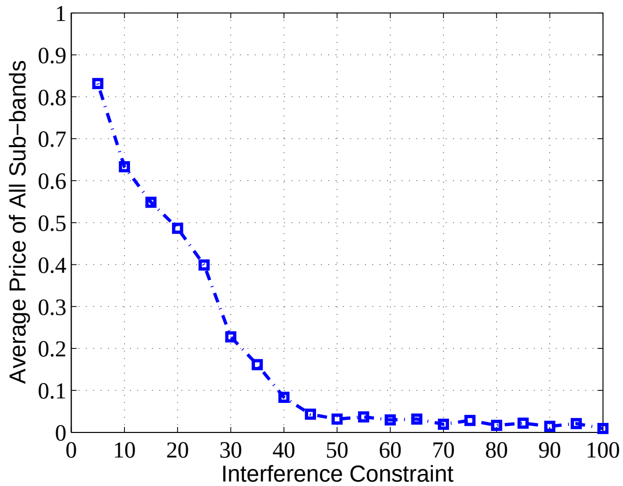

Fig. 8. The impacts of interference constraint: the average price $\overline{\mu}$ decreases with $\overline{Q}$.

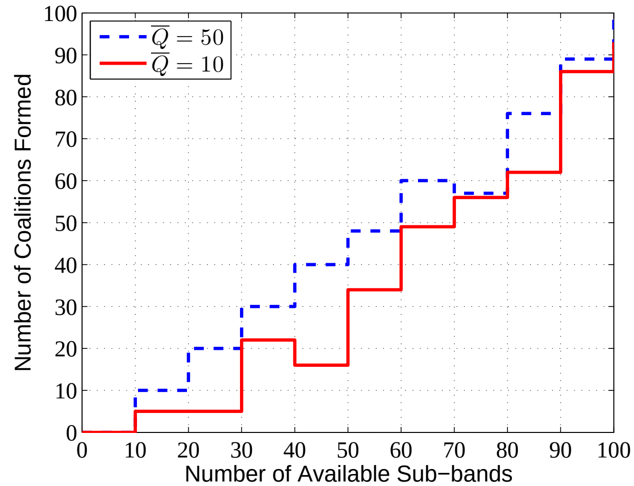

Fig. 10. Comparison of $\overline{Q} = 10$ and $\overline{Q} = 50$. The number of coalitions versus the number of sub-bands.

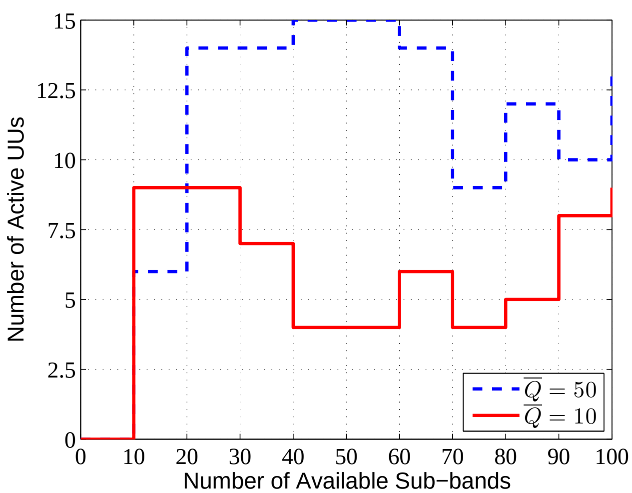

Fig. 9. Comparison of $\overline{Q} = 10$ and $\overline{Q} = 50$. The number of active UUs versus the number of sub-bands.

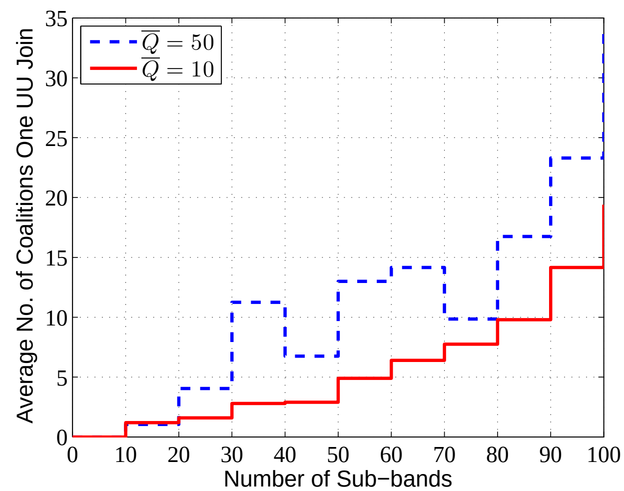

Fig. 11. Comparison of $\overline{Q} = 10$ and $\overline{Q} = 50$. The average number of coalitions one UUs join against the number of available sub-bands.

solution $p_{S_k}^{m*} = \left(\frac{1}{\mu^m h_{S_k}^m} - \frac{1}{\lambda_{S_k}^m}\right)^+$, it is seen that because $p_{S_k}^{m*}$ decreases with μ^m, in sub-band m, a larger transmit power $p_{S_k}^{m*}$ results in a smaller interference price μ^m.

Figs. 9 to 12 investigate the impact of the number of available sub-bands on the payoffs of UUs. Figs. 9 and 10 show the number of active UUs and the number of coalitions under different numbers of sub-bands, respectively. It is seen that the number of active UUs is always lower than the total number of UUs. The reason is that if the channel gains of some UUs are highly correlated, the low payoff UUs will always be forced to leave the coalition. From Fig. 9, it is observed that in general the larger $\overline{Q}$, the more active UUs because larger $\overline{Q}$ enables more chances for the UU to transmit. Fig. 10 shows that the more available sub-bands, the more coalitions formed because the number of coalitions is limited by the number of available sub-bands when overlapping is enable.

Figs. 11 and 12 show the average number of coalitions each UU joins and the average prices of sub-bands versus the number of sub-bands. Fig. 11 shows that the UU tends to join multiple coalitions when the number of available sub-bands increases, because in this case the players with lower pay-off in a crowded coalition may be better-off if joining a new coalition. Fig. 12 presents that the sub-band prices tend to decrease with the number of available sub-bands. When the UUs access multiple sub-bands, the aggregated interference in a single sub-band will be lower, which resulting in lower sub-band prices. Another observation is that the price at $\overline{Q} = 10$ is higher than that at $\overline{Q} = 50$ because the tolerated interference is low when $\overline{Q}$ is small. Therefore the price is accordingly higher.

Figure 13 compares state of the art coalition formation with the proposed overlapping one. It is illustrated directly that the improvement of data rate is achieved by enabling overlapping. When the power available for transmit is high, the UUs in OCF scheme are benefited by exploring more chances to transmit

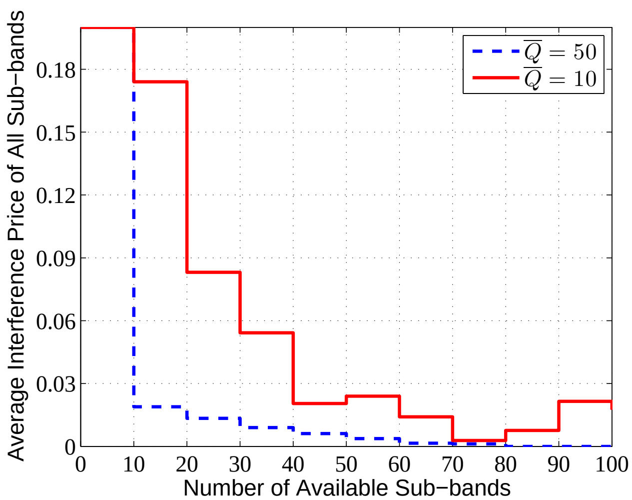

Fig. 12. Comparison of $\overline{Q} = 10$ and $\overline{Q} = 50$. The average interference prices versus number of sub-bands.

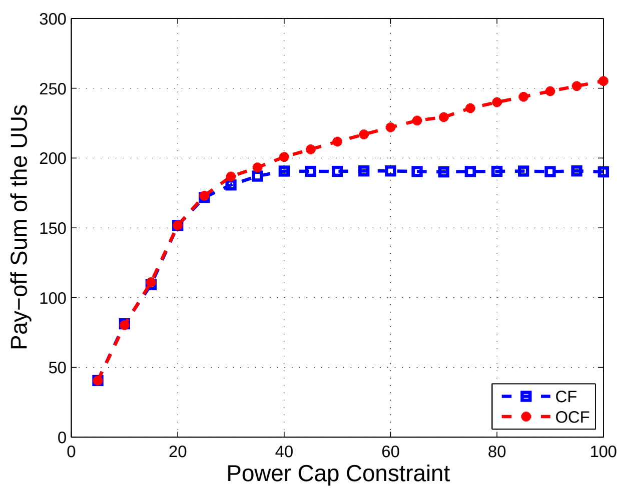

Fig. 13. The comparison between CF and OCF schemes.

on multiple sub-bands while in the CF schemes each of the UUs can only access a single sub-band.

VIII. Conclusion

The sub-band allocation and the power control issues in the carrier-aggregation-enabled HetNet are studied in this paper. We have developed a hierarchical game framework to jointly solve the power and sub-band allocation problems under the constraints of the transmit power and maximum tolerable interference level. A Stackelberg game is established for MCO to regulate the transmit power of the UUs so as to give sufficient protection to the LUs while optimizing the pay-off of the UUs. The OCF-game is also applied to analyze the behavior of the UUs that can self-organize into overlapping coalitions. We have proposed a simple two-layer algorithm for the UUs iteratively searching for the optimal coalition structure and the power allocations under different prices imposed by the macro-cell. It has been proved that the proposed algorithm can always converge to the SE of hierarchical game. At the same time the resulting transmit power and the sub-band allocation are stable and no players can further improve their payoff by unilaterally deviating from it by acting alone. Furthermore, by allowing the overlapping in the coalition formation among UUs, we have addressed the problem of sub-band and power allocation problem under two dimension constraints. The proposed framework can also be extended into more general network setting with multiple BSs to cooperatively share their sub-bands or the downlink communication that multiple LUs need to be protected.

Appendix A
Proof of Proposition 1

Suppose that a partial coalition $\boldsymbol{p}^{m*} = \{p_{S_k}^{m*} : k = 1, 2, ...K\}$ is formed on sub-band m, in which the positive power $p_{S_k}^{m*}$ is given by (13), i.e.,

$$\boldsymbol{p}^{m*} = \arg\max_{\boldsymbol{p}^m} \pi(\boldsymbol{p}^m). \tag{26}$$

We define the support of $\boldsymbol{p}^{m*}$ as,

$$\text{supp}(\boldsymbol{p}^{m*}) = \{S_k : p_{S_k}^{m*} > 0, k = 1, 2, ...K\}^m, \tag{27}$$

which defines a coalition of UUs regardless the resource distribution. Hence, for any other partial coalition $\boldsymbol{p}^{m'}$ with the support supp$(\boldsymbol{p}^{m*})$, we have

$$\pi(\boldsymbol{p}^{m*}) \geq \pi(\boldsymbol{p}^{m'}), \tag{28}$$

i.e., the partial coalition $\boldsymbol{p}^{m*}$ *blocks* all other partial coalitions formed on sub-band m which involves with supp$(\boldsymbol{p}^{m*})$.

Therefore, we can say that the partial coalition $\boldsymbol{p}^{m*}$ in our proposed game is one-to-one correspondent to the coalition $\{S_k : p_{S_k}^{m*} > 0, k = 1, 2, ...K\}^m$ formed on sub-band m. Since $\{S_k\}^m \subseteq \mathcal{K}$, i.e., $\{S_k\}^m$ is a subset of $\mathcal{K}$, the number of all possible partial coalitions equals to the number of subset of $\mathcal{K}$, which is given by,

$$\sum_{n=1}^{K} \binom{K}{n} = 2^K - 1. \tag{29}$$

This concludes the proof. $\square$

Appendix B
Proof of Proposition 2

1) Continuous. The value function in (22) is the difference between a log function and a linear function, which is obviously continuous.
2) Monotone. The interference power constraint in (1) limits the total transmit power allocated in sub-band m indirectly by pricing in the Stackelberg game. Hence the power allocated by S_k in sub-band m is bounded by $p_{S_k}^{m*}$. Since the pay-off function, $\pi(p_{S_k}^m)$, of S_k is concave, then for any $\pi(p_{S_k}^{m'}) \in [0, p_{S_k}^{m*}]$ we have $\pi(p_{S_k}^{m'}) \leq \pi(p_{S_k}^{m*})$. Therefore for any $\boldsymbol{p}^{m'}$ and $\boldsymbol{p}^{m*}$, such that $p_{S_k}^{m'} \leq p_{S_k}^{m*}$, we have $\boldsymbol{v}(\boldsymbol{p}^{m'}) \leq \boldsymbol{v}(\boldsymbol{p}^{m*})$, i.e., the value function is monotone.

3) Bounded. According to the proof in 2), the value function is bounded by $\boldsymbol{v}(\boldsymbol{p}^{m*})$, where $\boldsymbol{p}^{m*} = (p^m_{S_1}, p^m_{S_2}, ..., p^m_{S_K})$ satisfies $\sum_{k=1}^{K} h^m_{S_k} p^m_{S_k} = \overline{Q}$.
4) U-finite. The proof can be referred to proposition 1.
5) The inequality. The equality of (24) is always taken in the proposed game since the value function is the summation of individual pay-off of the member players.

□

Appendix C
Proof of Proposition 3

In previous section we proved that finding optimal pricing using

$$\mu^{m*} = \arg\max_{\mu^m} \pi_{MCO}(\boldsymbol{p}^*, \mu^m)$$

is equivalent to solving

$$\sum_{k=1}^{K} \left(\frac{1}{\mu^{m*} h^m_{S_k}} - \frac{1}{\lambda^m_{S_k}} \right)^+ h^m_{S_k} = \overline{Q}.$$

Hence the pay-off maximizing for MCO can be achieved by choosing the optimal price to control the interference approaching $\overline{Q}$. In other words, the only two cases that the MCO will stop further increasing or decreasing prices are, 1) $\sum_{k=1}^{K} p^m_{S_k} h^m_{S_k} \leq \overline{Q}$, and 2) $\mu^m = 0$. In other words, the price μ^m can always converge to a fixed price. □

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